The Franck-Hertz Experiment - A Quantum Effect

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Abstract

This laboratory demonstrates the discrete energy levels in an atom, proving the quantum nature in atomic scale. The sequential peaks and dips in the current are observed as the voltage increase, indicating distinct levels of excited states. When electrons are heated and accelerated to 4.9720 ± 0.0002 eV, they undergo inelastic collisions with vaporized mercury atoms, leading to the excitation of the ground state electrons of the mercury atom to the excited states. As the electrons transition back to their ground state, they emit light with a wavelength of 249.54 ± 0.02 nm.

1 Introduction

The Franck-Hertz experiment, first performed by the German physicists James Franck and Gustav Hertz in 1914, marked a milestone in the validation of the Bohr model. It effectively demonstrated the discrete energy levels in an atom, establishing foundation for quantum physics.

In the experiment, heated electrons undergo collisions with vaporized mercury gas. At non-specific kinetic energy, electrons elastically collide with mercury atoms due to the substantial mass difference between the electron and mercury. However, upon reaching a specific kinetic energy, electrons inelastically collide with mercury atoms, exciting the ground state electron of the mercury atom to the excited state. Subsequent to this excitation, when the electron transitions back to the ground state, it emits light with an wavelength of 253.6 nm and energy of 4.9 eV [1].

To determine the wavelength and photon energy of the emitted light, two key equations are applied:

$$E = eV \tag{1}$$

$$\lambda = \frac{hc}{E} \tag{2}$$

where E represents the kinetic energy transferred to mercury atom in electron volt, e is the elementary charge constant, V represents the acceleration voltage, h is Planck's constant, λ represents the wavelength of emitted photons, and c is the speed of light.

2 Materials and Methods

2.1 Materials

Apparatus (refer to figure 1 for the experimental set-up):

- Four DC Voltages Sources, where:
  - E1 is the filament supply,
  - E2 is the screen grid voltage,
  - E3 is the accelerating voltage
  - E4 is the fixed voltage (to repel the low energy electrons)
- Frank-Hertz tube containing mercury filled vapor and low pressure gas.
- Oven Heater ($\pm 0.5^{\circ}$ C)
- Voltmeter (± 0.0005 V)
- The Keithley Electrometer
- FranckHertz.vi Data Collection Software

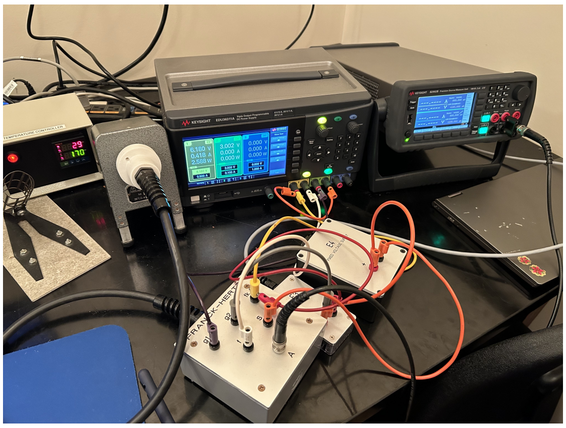

Figure 1: Experimental Setup

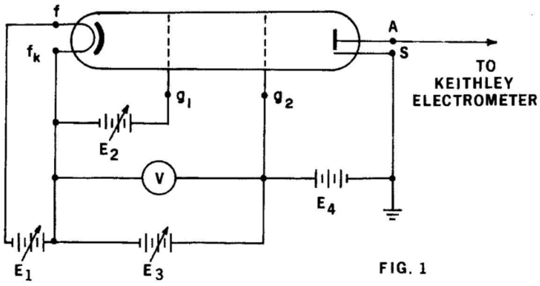

Figure 2: The Frack-Hertz Apparatus [2]

2.2 Methods

Preparing the apparatus is a crucial step before data collection. Before activating any of the four power supplies, heat the tube and oven to the operational temperature of 170 degrees Celsius, ensuring that the tube temperature never exceeds 210 degrees Celsius.

Next, set the E1 voltage source for the filament supply to 6 volts, and connect the E2 voltage source for the screen grid voltage to 2 volts. Connect the E3 voltage source for the accelerating voltage to the X computer ports from the instrument panel, and link the Keithley electrometer to the Y computer ports.

Once the apparatus is prepared, launch the *FranckHertz.vi* on the desktop. Click on "Clear Graph" if needed, and then click "STOP." Switch the Sweep Circuit to "RESET" and then to "RUN" to start recording. After data collection, click "STOP" and switch to "RESET" to zero the electrometer.

Furthermore, highlight the end-of-scale number and modify it to another value if necessary to change the graph axis. The positions of relative maxima/minima can be determined using the two cursors.

3 Data and Analysis

The experiment was repeated four times to minimize the impact of uncertainties and to evaluate the precision of the results obtained. The scatter plot of the Electrometer Current (A) vs. Accelerating Voltage (V) of trial 1 is shown in figure 3. The scatter plots for the remaining trials are attached in section 7, Appendix A. The experiment was repeated with varying values of the E2 DC Voltage source, and subsequently E2 = 2V yielded the optimal results with minimal number of anomalies (i.e. sudden or sharp jumps or drops in the otherwise continuous and repeating pattern with the voltage and current) in plotted scatter graph. As such trials 1-4 were conducted at $E2 = 2$ volts.

The essential calculation to make from the graphs in section 7 are voltage differences occuring at peaks of this graph. The ΔV values were experimentally determined by finding the voltage difference between accelerating voltages of Channel 2 (Source) that occurs at the two consecutive peaks/maxima of the electrometer current. This can be expressed using the equation:

$$\Delta V_i = V_{n+1} - V_n \tag{3}$$

where V_{n+1} and V_n represent the voltages at the consecutive peaks n and $n+1$, $n \geq 0$. For example in trial 1 the first voltage difference value between peak 0 and peak 1 is computed using:

$$\Delta V = 11.3920\ V - 5.5410\ V = 4.8510\ V \tag{4}$$

The processed values for ΔV have been summarized in table 1. A total of 7 peaks were observed for each trial and as such there were 6 voltage difference ΔV values for each trial. Peak 0 is a the preliminary measurement and therefore there is no associated ΔV with it and has been indicated with '-'. The average voltage difference value is computed using the standard formula for mean using:

$$\bar{x}_{est} = \frac{\sum_{i=1}^{N} x_i}{N} \tag{5}$$

The ΔV_{avg} computed for each trial will be used to compute the energy transferred to Hg atom and the wavelength of emitted light.

Table 1: The Accelerating Voltage (V) and Voltage Differences for four trials

(a) ***Trial 1*** (refer to Appendix A, figure 4)

Maxima Number	Voltage (V) $\pm 0.0005 V$	ΔV ($\pm 0.0007\ V$)
0	6.5410	-
1	11.3920	4.8510
2	16.4190	5.0270
3	21.4220	5.0030
4	26.5270	5.1050
5	31.7460	5.2190
6	36.9320	4.6460
ΔV_{avg}	$4.9750 \pm 0.0003\ V$	

(b) ***Trial 2*** (refer to Appendix A, figure 5)

Maxima Number	Voltage (V) ($\pm 0.0005 V$)	ΔV ($\pm 0.0007\ V$)
0	6.6670	-
1	11.4230	4.7560
2	16.4720	5.0490
3	21.2540	4.7820
4	26.1030	4.8490
5	31.0640	4.9610
6	36.3890	5.3250
ΔV_{avg}	$4.9530 \pm 0.0003\ V$	

(c) ***Trial 3*** (refer to Appendix A, figure 6)

Maxima Number	Voltage (V) ($\pm 0.0005\ V$)	ΔV ($\pm 0.0007\ V$)
0	6.5520	-
1	11.3450	4.7930
2	16.2950	4.9500
3	21.1630	4.8980
4	26.1650	5.0020
5	31.3620	5.1970
6	36.4780	5.1160
ΔV_{avg}	$4.9880 \pm 0.0003\ V$	

(d) ***Trial 4*** (refer to Appendix A, figure 7)

Maxima Number	Voltage (V) ($\pm 0.0005\ V$)	ΔV ($\pm 0.0007\ V$)
0	6.6410	-
1	11.4270	4.7860
2	16.2290	4.8020
3	21.1660	4.9370
4	26.2810	5.1150
5	31.2850	5.0040
6	36.4760	5.1910
ΔV_{avg}	$4.9730 \pm 0.0003\ V$	

The scatter plots shown in figure 3 and in section 7 exhibit a non-sharp saw-tooth pattern. Scientifically, inside the accelerating apparatus, accelerated electrons are emitted from the filament and collected at the cathode. If electrons undergo elastic collisions, they will pass through the cathode. On the other hand, electrons undergoing inelastic collisions will lose a specific amount of energy by exciting the electrons in the mercury atom and will be collected at the cathode with lower kinetic energy. This results in lower current readings in electrometer since the current is proportional to the number of electrons reaching the cathode, which depends on the accelerating voltage. Referring to figure 3 and table 1, dips are observed sequentially at intervals of average of 4.9750 ± 0.0003, 4.9530 ± 0.0003, 4.9880 ± 0.0003, and 4.9730 ± 0.0003 volts respectively.

4 Discussion

4.1 Energy Transferred from an Electron

The amount of energy (in eV) transferred from an electron to an Hg atom in an inelastic collision can be determined using **equation 1**. **Table 2**, below, shows the expected value (mean) value of the energy transferred (eV) for each of the trials. Since $E = eV$, and the elementary charge, e, is a constant and therefore does not have an associated uncertainty, the uncertainty in E is equivalent to the uncertainty in the voltage differences (ΔV_{avg}).

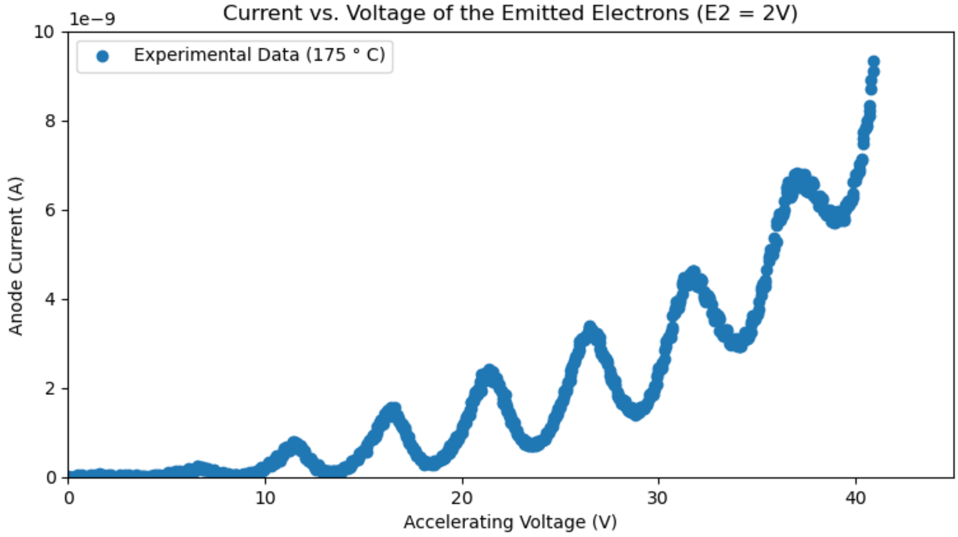

Figure 3: Franck-Hertz (Current Vs. Voltage) Scatter Plot - Trial 1. The error bars in the accelerating voltage and anode current derives from the uncertainties in data collection software and they have been incorporated on to the graph. Note however these errors are to small to be visible.

For example for trial 1 in table **1a**, the ΔV_{avg} value is $4.9980 \pm 0.0003\ V$. This means that the amount of energy transferred from the electron to Hg atom is correspondingly $4.9980 \pm 0.0003\ eV$. The mean E value of the amount of energy transferred was determined by taking an average of the 4 trials. The propagated uncertainty in the mean E was determined using equation 8. The mean E value was determined to be $4.9720 \pm 0.0002\ eV$. The statistical error in the mean E is determined using equation 10 which equals $0.0145\ eV$. The statistical error is relatively larger than the measured uncertainty, indicating presence of random errors plausibly due to fluctuations in oven temperature.

4.2 Wavelength of Emitted Photons

The wavelength associated with photons emitted by Hg atoms when it decayed from the first excited state to the ground state can be determined using equation 2. The uncertainty in wavelength is propagated using equation 11.

Table 2: The Amount of Energy (eV) transferred and the Wavelength (nm) of the emitted photons

Attempt	Energy, E ($\pm 0.0003\ eV$)	Wavelength, λ	Uncertainty ($\delta\lambda$)
Trial 1	4.9750	249.39	$\pm$ 0.02
Trial 2	4.9530	250.48	$\pm$ 0.02
Trial 3	4.9880	248.77	$\pm$ 0.02
Trial 4	4.9730	249.53	$\pm$ 0.02
Mean E	4.9720 $\pm 0.0002\ eV$	Mean λ	$249.54 \pm 0.02\ nm$
Statistical Error (σ_E)	$0.0145\ eV$	Statistical Error (σ_λ)	$0.7\ nm$

4.3 Comparison of the mean E and λ to the literature value

The mean value of E and λ was $4.9720 \pm 0.0002\ eV$ and $249.54 \pm 0.02\ nm$ respectively. The literature value of E in the original Frank-Hertz Experiment is $4.9\ eV$ [1], which lies outside the range of the experimental value's uncertainty (of $0.0002\ eV$). Neither does λ value lie in the range of the uncertainty. Similarly, the literature value for the energy and wavelength of the emitted light is also outside the range of the the statistical error (i.e. standard deviation). This difference in the literature and experimental value exists because of several sources of errors that impact the accuracy of the results obtained. Firstly, inaccuracies and variations in the heating of the FrankHertz tube is a significant source of error. An insufficient heating or overheating of the Frank-Hertz tube could result in more inaccuracies in reading and identifying the minima/maxima of the anode current. Overheating will result in a significant reduction in the emission current, whilst insufficient heating will result in a significant increase the emission current.

4.4 Vaporized Mercury Gas

Franck and Hertz used vaporized mercury over hydrogen gas for two reasons. First, due to the nature of this experiment, it is necessary for the electrons to collide with substantially massive element to facilitate elastic collision when it is below the threshold temperature. Comparing atomic masses, mercury, with the mass of 200.59 u far exceeds the mass of hydrogen, which is 1.0078 u. Second, the experiment has to be performed by using monatonic gas. As electrons interact with the gas, there is a possibility of energy transfer occurring at the molecular level, leading to the formation of binding bonds. To minimize such reactions, it is beneficial to use monatonic gas in which gas particles do not form bonds with each other [4]. Other noble gases such as Argon and Neon are also viable alternatives for this experiment. Therefore, vaporized mercury gas is the optimal choice of element, satisfing both conditions.

4.5 Non-Sharp Sawtooth Patterns

The dips observed in the current versus accelerating voltage graph [figure] do not exhibit perfectly sharp sawtooth patterns, due to the distribution of velocities of the thermionically emitted electrons and the rise of the excitation cross section [4].

First, thermionic emission is the process of electrons being emitted from the heated source. Since the electrons are heated up to a certain temperature, some electrons may possess varying initial energies. This results in a diverse range of kinetic energies when the voltage is applied. Second, the increased excitation cross section indicates more randomly distributed mercury atoms. Therefore some electrons may undergo multiple collisions, losing more kinetic energy upon reaching the cathode. Consequently, these two factors contribute to a broader range of kinetic energy losses, leading to less sharp, more extended dips in the observed results.

5 Uncertainty Propagation

5.1 Uncertainty in ΔV

To propagate the uncertainties in the ΔV values shown in **Table 1**, the propagation formula for addition/subtraction can be utilized. Therefore, the uncertainty in ΔV is given by [3]:

$$\delta(\Delta V) = \sqrt{(\delta V_1)^2 + (\delta V_2)^2} \tag{6}$$

where δV_1 and δV_2 are the experimental uncertainties in the accelerating voltages of the two consecutive maximas V_1 and V_2. The largest source of uncertainty in this experiment derives from the precision of significant figures reported by the *FrankHertz.vi* Data Collection software. Since the uncertainty in V_1 and V_2 are both $\pm 0.0005\ V$, the final uncertainty in ΔV is:

$$\delta \Delta V = \sqrt{(0.0005)^2 + (0.0005)^2} = \sqrt{2}(0.0005) \approx 0.0007\ V \tag{7}$$

5.2 Uncertainty in Mean

The uncertainty in ΔV_{avg} can be computed using the standard mean error formula given by [3]:

$$\delta \bar{X}_{est} = \frac{\delta X}{\sqrt{N}} \tag{8}$$

where $\delta \bar{X}_{est}$ is the uncertainty in the estimated mean, N is the number of quantities, and δX is the uncertainty in the measured reading. For trials 1-4, shown in 1, since there are 7 peaks observed in the accelerating voltage vs. current graph, there are 6 ΔV values. As such, N equals 6 because the average of each ΔV measurement observed on the graph is accounted for. Using equation 8, the uncertainty in ΔV_{avg} calculated in Table 1a for instance, is:

$$\delta(\Delta V_{avg}) = \frac{0.0007}{\sqrt{5}} = 0.0003^1 \tag{9}$$

It is important to acknowledge that the uncertainty in the voltage readings are relatively small because the all instruments were calibrated correctly, meaning there was no non-zero offset error in neither the voltmeter or the electrometer. Additionally FrankHertz.vi Data Collection software recorded extremely precise readings with upto 4 decimal places in the voltage readings which means that the instrumental uncertainties are extremely small.

5.3 Statistical Error

The statistical error in the estimated value of λ and E can be determined using the standard deviation formula expressed using equation 10. Statistical error is utilized because the instrumental uncertainty is too small to reflect the difference in the calculated and the literature values of λ and E.

$$\sigma_{est} = \sqrt{\frac{\sum_{i=1}^{N}(x_i - \bar{x}_{est})^2}{N-1}} \tag{10}$$

where N is the number of trials, x_i is the measured value, $\bar{x}_{est}$ is the mean value [3]. Using equation 10 the statistical error in the mean E is $\sigma = 0.0145\ eV$ and the statistical error in the mean λ is $\sigma = 0.7\ nm$. It is important to acknowledge the propagated uncertainties (from the apparatus) is significantly smaller than the statistical error which indicates the presence of random errors within the experiment.

5.4 Uncertainty in λ

The value for λ can be computed using $\lambda = \frac{hc}{E}$. Since the planck's constant, h, and speed of light, c, are fundamental constants, they do not have associated uncertainties. As such, the uncertainty in λ is only dependent upon the fractional uncertainty in E and can be expressed as:

$$\delta\lambda = \lambda\sqrt{\frac{(\delta E)^2}{E^2}} = \lambda\frac{\delta E}{E} \tag{11}$$

For example for the first trial with an energy value of $4.9750 \pm 0.0003\ eV$ and a wavelength of $249.39\ nm$ has an uncertainty:

$$\delta\lambda = 249.39\ [nm] \times \frac{0.0003}{4.9750} \approx 0.02 nm \tag{12}$$

6 Conclusion

In conclusion, this laboratory experiment proficiently illustrates the quantization of energy levels in the mercury atom. The determined mean value for the emitted photon's electron voltage and wavelength were $4.9720 \pm 0.0002\ eV$ and $249.54 \pm 0.02\ nm$, respectively. It corresponds to the required energy that accelerated electrons must possess to excite electrons in the mercury atom. Thus, by combining theoretical principles, this laboratory effectively replicates the outcomes of the original Franck-Hertz experiment, an energy of $4.9\ eV$ and a wavelength of $253.6\ nm$ [1].

References

[1] G. Hon and B. R. Goldstein, "Centenary of the Franck-Hertz experiments," *Annalen der Physik*, vol. 525, no. 12, pp. A179–A183, 2013, doi: 10.1002/andp.201300744.

[2] PHY294, *The Franck-Hertz Experiment*, 2024.

[3] PHY294, *Error Analysis for 2nd Year Experimental Physics*, 2024.

[4] A. C. Melissinos, *Experiments in modern physics*, 2nd ed. San Diego: Academic Press, 2003.

[1] This also reflects the uncertainties in the estimated mean computed for Trials 2, 3, and 4 because the uncertainty in ΔV are the same for all measurements, $\delta(\Delta V) = 0.0007\ V$.

7 Appendix A

Figures 3, 4, 5, and 6 display the scatter plots for the experimental trials 1, 2, 3, 4 respectively. Note: the data obtained in the *FrankHertz.vi* data collection software was exported to a python program to sketch the graph. This graph does not include a best fitting curve because this plot is not expected to follow a defined fitting function/model or a mathematical relationship. The uncertainty in the anode current is $\pm 5 \times 10^{-13}$ because the CH1 Current values recorded by the data collection were accurate up 12 significant figures. Similarly the uncertainty in the voltage is $\pm 0.0005\ V$ because the data collection software quotes the values of the ramping voltage with an accuracy of 3 decimal places.

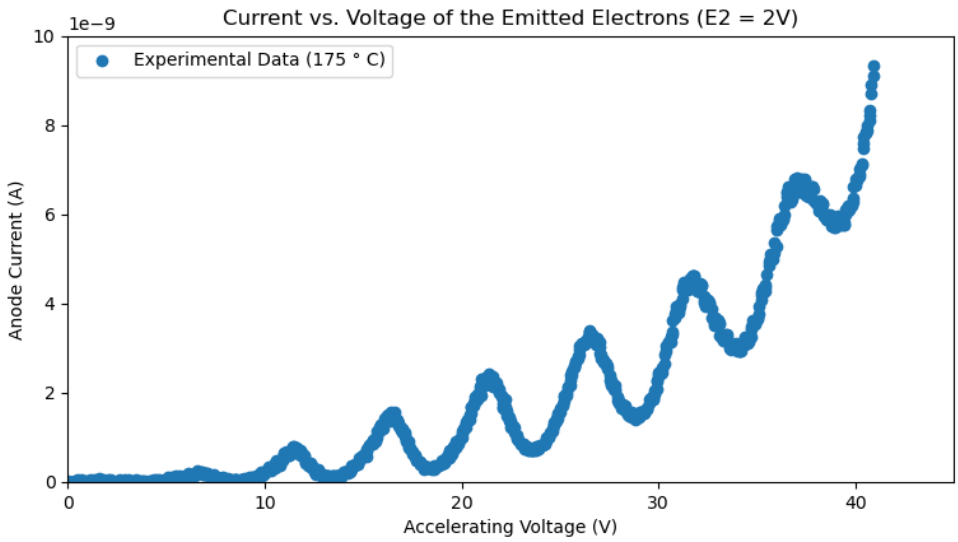

Figure 4: *Trial 1* for E2 = 2 Volts at temperature 170°C

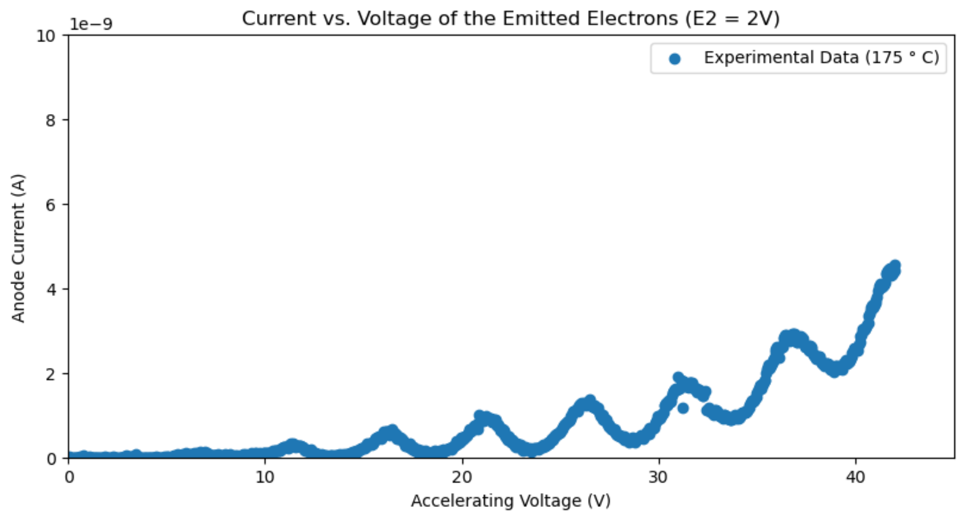

Figure 5: *Trial 1* for E2 = 2 Volts at temperature 170°C

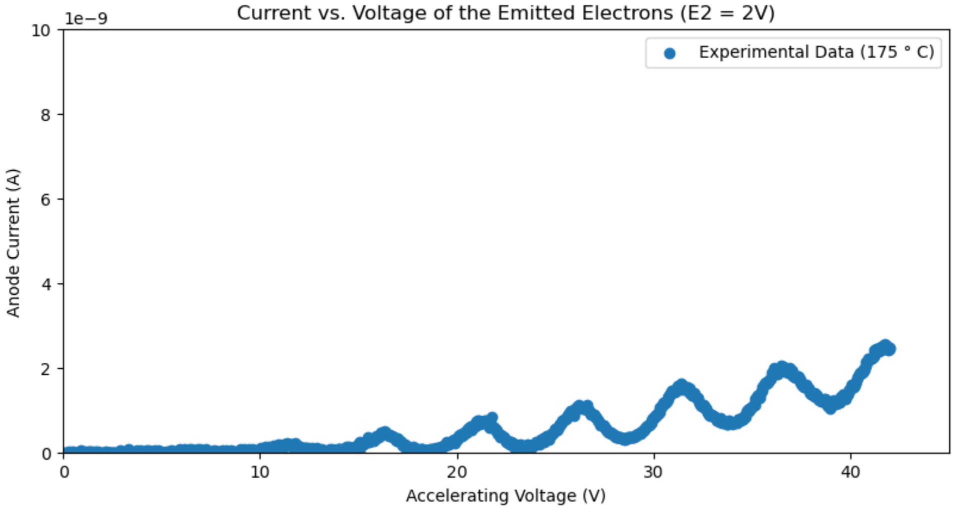

Figure 6: *Trial 1* for E2 = 2 Volts at temperature 170°C

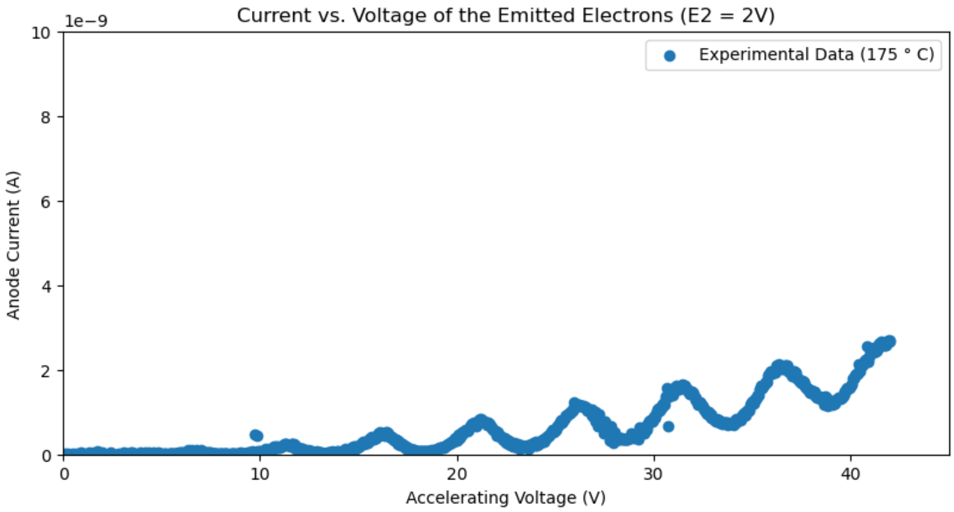

Figure 7: *Trial 1* for E2 = 2 Volts at temperature 170°C